Calculus II | Homework 0421 | Name:
Dr. Paul L. Bailey | Tuesday, April 20, 2021

Due Wednesday, April 21.

**Proposition 1.** **($n^{\text{th}}$ Term Test)**
*If $\sum a_n$ converges, then $a_n \to 0$.*

**Proposition 2.** **(Alternating Series Test)**
*An* alternating series *is a series of the form*

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + \cdots ,$$

*where $u_n > 0$ for all $n \in \mathbb{N}$.*

*The series $\sum (-1)^n u_n$ converges if all three of these conditions are satisfied:*

**1.** $u_n > 0$ *for all* $n \in \mathbb{N}$.

**2.** $u_n \geq u_{n+1}$ *for all* $n \in \mathbb{N}$.

**3.** $u_n \to 0$.

**Problem 1.** Use the $n^{\text{th}}$ Term Test or the Alternating Series Test to determine if the following series converge. Write your findings in a complete sentence which justifies your conclusion.

**(a)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

**(b)** $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln(n)}{\ln(n^2)}$

**Proposition 3. (Integral Test)**
*Suppose that* $a_n = f(n)$ *for all* $n \in \mathbb{N}$*, where* $f$ *is continuous, positive, and decreasing. Then*

$$\sum_{n=1}^{\infty} a_n \text{ converges} \quad \text{if and only if} \quad \int_1^{\infty} f(x)\,dx \text{ converges.}$$

**Problem 2. ($p$-Series Test)**
Use the Integral Test to show that

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges} \quad \text{if and only if} \quad p > 1.$$

**Proposition 4. (Limit Comparison Test)**
*Let* $a_n, b_n \leq 0$ *for all* $n \in \mathbb{N}$*. Let* $\lambda = \lim \frac{a_n}{b_n}$*. Then*

- *If* $0 < \lambda < \infty$*, then* $\sum a_n$ *and* $\sum b_n$ *either both converge, or both diverge.*
- *If* $\lambda = 0$ *and* $\sum b_n$ *converges, then* $\sum a_n$ *converges.*
- *If* $\lambda = \infty$ *and* $\sum b_n$ *diverges, then* $\sum a_n$ *diverges.*

**Problem 3.** Use the Limit Comparison Test to determine if the following series converge. Write your findings in a complete sentence which justifies your conclusion.

**(a)** $\sum_{n=2}^{\infty} \frac{n}{n^2 - 2}$ (compare to $\sum \frac{1}{n}$)

**(b)** $\sum_{n=2}^{\infty} \frac{n}{n^3 - 2}$ (compare to $\sum \frac{1}{n^2}$)